triangle. You say, p. 624: "Now the angle MEN being a *w*-angle equals the angle $\beta\epsilon\delta$..."; in other words, you assume that the size of the isosceles right triangle has no effect on the size of its acute angles. This is Wallis's form of the parallel postulate. And so you are guilty of begging the question.

Not only would a smattering of Bolyai have saved you but so would a little excursion into Chapter XV of my *Rational Geometry,* which starts by saying: "Deducing spherics from a set of assumptions which give no parallels, *no similar figures,* we get a two-dimensional non-Euclidean geometry, yet one whose results are also part of three-dimensional Euclidean."

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A REMARK ON F. C. RUSSELL'S THEOREM.

F. C. Russell of Chicago has endeavored in the April number of *The Monist* to disprove the legitimacy of the non-Euclidean geometry by showing the demonstrability of the parallel postulate. The basis of his considerations has been laid on a simple proposition that the angle-sum of an isosceles right-angled triangle equals two right angles; a proposition the proof of which he does not dare give, saying it would be "spreading an imputation upon the reader," being so simple in nature. But the whole secret of the matter remained concealed under this unknown sort of a proof, and so we are lucky that we had it imparted to us by Russell himself in a subsequent number. In studying it, we have found all that can be desired.

Russell defines his *u*-angles "as being such angles as the sides of an isosceles right-angled triangle make with the hypotenuse." This definition is of course not in any way objectionable, but when Russell has to consider the *u*-angles arising from different triangles of unequal sizes, to be always equal, he has unconsciously fallen into a pit of thought, from which he is unable to get out. When we adhere to the Euclidean world, we can well prove the assumption Russell makes, but how can he protest the legitimacy of it, when he is going to show the Euclidean system to be the sole one that can be relied upon? If he wants to be credited by us, he must first prove the assumption he has made; which most probably he cannot do without having recourse to the parallel postulate or some-

thing else that may be substituted for it. In a word, Russell has substituted a different axiom in place of the postulate of Euclid. His endeavor and achievement have however left nothing that could make a step towards disarming the pan-geometricians. We stand uninjured on the same ground as before in spite of all the desperate assaults from the strong hand of Russell, who has utterly failed to disground us.

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A MATHEMATICAL PARADOX.

The following paradox appears to me to be interesting because it shows how "common sense" breaks down when dealing with a slightly subtle question.

The question to be discussed is: Is the greatest weight that a man can lift the same as the least weight that he cannot lift, or not; and if the weights are different, which is the greater?

The numerical values of all possible weights (both those which the man A can, and those which he cannot, lift at the particular moment considered) form the simply-ordered aggregate of positive real numbers R. Those weights that A can (at this particular time) lift bring about what Dedekind* called a section (*Schnitt*) in R, and all the members of R fall into the two classes:

a. The class of those numbers x such that A can lift the weight x (then also A can lift any of the weights less than x);

b. The class of those numbers y such that A cannot lift the weight y (then also A cannot lift any of the weights greater than y).

Now, as is well known, there is one, and only one, number which "generates" this section, and this number is either the upper limit of the class (a), or the lower limit of the class (b), but not both.

Thus, our answer to the question about the weights is: *Either* there is a greatest weight that a man can lift, *or* there is a least weight that he cannot lift, *but not both.* The paradox lies in the fact that, to unaided common sense, the existence of a limit seems just as, or even more, plausible in both cases or neither as in one

* *Stetigkeit und irrationale Zahlen,* Braunschweig, 1872 and 1892 (English translation in Dedekind's *Essays on the Theory of Number,* Open Court Publishing Co., Chicago, 1901).